System Identification with Perfect Sequences Based on the NLMS Algorithm

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Dedicated to Professor Hans Dieter Luke on the occasion of his 60th birthday

System Identification through Arrangement of the NLMS-Algorithmus mittels perfekter Sequenzen

The problem of determining the impulse response of an unknown linear time-invariant system has been studied in many papers. One approach for the identification problem is the so-called Fast M-sequence transform, where a binary periodic maximal-length sequence is used as excitation signal for the unknown system. This identification process bases on the cross-correlation between one period of the maximal-length sequence and its received system response.

In this paper an alternative to several existing approaches is proposed by using a periodic perfect sequence as stimulus signal for the normalized least mean square (NLMS) algorithm. In contrast to the commonly used stochastic white noise processes with this deterministic excitation the NLMS algorithm is capable of identifying a linear noiseless system within one period. Several theoretical aspects of this technique will be discussed considering that in many applications such as acoustic echo cancellation or active noise control the NLMS algorithm is applied, the practical point of view of the proposed method becomes apparent.

Keywords: System identification, adaptive filters, NLMS algorithm.

1. Introduction

Several approaches have been proposed to the problem of system identification. For the measurement of an electro-acoustic echo path, for instance, the so called Fast M-sequence transform was developed as e.g. [1]-[3] based on the cross correlation between a special stimulus signal and its system response. In this contribution a related approach, the normalized least mean square (NLMS) algorithm [4] using a special excitation signal, is introduced.

The discrete time model in Fig. 1 describes the identification process of an unknown linear transmission system by applying an adaptive filter. The unknown transmission path is represented by the impulse response $g = (g_0, g_1, \ldots, g_N-1)^T$ of length $N$. At the output of the unknown system for simplicity a stationary white noise signal $n(t)$ is added to consider the influence of background noise on the adaptation process.

The adaptation of the digital filter coefficients is driven by the NLMS algorithm, i.e. the weights of the adaptive filter are controlled by the iterative equation

$$e(k+1) = e(k) - \alpha \hat{f}(k) p(k)^T$$

(1)

with the stepsize $\alpha$ and the difference signal

$$\hat{f}(k) = [g - e(k)] p(k) + n(k)$$

(2)

where

$$e(k) = (e_0, e_1, \ldots, e_{N-1})^T$$

$$p(k) = (p_0, p_1, \ldots, p_{N-1} - N + 1)^T$$

$$\hat{f}(k) = p(k)^T p(k)$$

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2. System Identification

2.1 Identification of an LTI System

This section contains the proof that the perfect sequence excitation leads to an exact system identification within \( N \) iterations. For the following derivations it is assumed that

- the unknown system is linear and time-invariant (LTI),
- the system is noiseless, i.e. \( n(i) = 0 \),
- stepsize factor is \( \alpha = 1 \) and
- the adaptive filter and the unknown system are of same length \( N \).

For these assumptions the combination of eqs. (1) and (3) leads to

\[
c(i + 1) = c(i) + p^T(i) \frac{[g - c(i)]}{||p(i)||^2} p(i). \tag{6}
\]

The multiplication of eq. (6) with \( p^T(j) \) results in

\[
p^T(j) c(i + 1) = \begin{cases} p^T(i) g & j = i \\ p^T(j) c(i) & j \neq i. \end{cases}
\]

Due to the special correlation properties of perfect sequences \( N \) consecutive vectors \( p(i), p(i+1), \ldots, p(i+N-1) \) represent an orthogonal basis in the \( N \)-dimensional vector space. Consequently, above equation illustrates that the \( i \)-th component of vector \( c(i + 1) \) and \( g \) do match, while the remaining \( N - 1 \) components are still unchanged. During the adaptation process in each iteration one component of vector \( g \) is identified. After \( N \) iterations equation

\[
c(i + N) = g \tag{8}
\]

is obtained. So independently of the initial set of coefficients \( c(i) \) the impulse response of the unknown LTI system is identified after \( N \) iterations.

In order to verify above consideration a perfect sequence and a spectrally white noise signal were applied as input signal \( p(i) \) to the system given in Fig. 1. The results are summarized in Fig. 2 in terms of the (logarithmic) system distance \( D(i) \), which is introduced as a measure of quality according to

\[
D(i) \text{ dB} = 10 \log \frac{||g - c(i)||^2}{||g||^2}. \tag{9}
\]

Applying the perfect sequence periodically to the input of the system provides the expected optimal results, i.e. perfect sequences represent the optimal excitation signal of the NLMS algorithm. According to Fig. 2 the initialization phase takes \( 2N \) iterations to achieve perfect adaptation. This delay is caused by the \( N \) empty filter filter states of the unknown system and by \( N \) iterations, which are required to adapt \( N \) filter coefficients. Consequently, within the continuous identification process only \( N \) iterations are required.

However, especially the comparison with the system distance achieved with a white noise excitation as e.g. used in [5], [6] visualizes the advantage of the proposed
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- stepsize factor is $\alpha = 1$, and
- the adaptive filter and the unknown system are of same length $N$.

For these assumptions the combination of eqs. (1) and (3) leads to
$$e(i + 1) = e(i) + \frac{p^*(j) [g - c(j)]}{|p(i)|^2} \cdot p(i).$$

(6)

The multiplication of eq. (6) with $p^*(j)$ results in
$$p^*(j) e(i + 1) = \left( p^*(j) p(j) \right) \frac{g}{\| p(j) \|^2} \cdot e(j) + j \neq i,$n

(7)

Due to the special correlation properties of perfect sequences $N$ consecutive vectors $p(i), p(i + 1), \ldots, p(i + N - 1)$ represent an orthogonal basis in the $N$-dimensional vector space. Consequently, above equation illustrates that the $i$-th component of vector $e(i + 1)$ and $g$ do match, while the remaining $N - 1$ components are still unchanged. During the adaptation process in each iteration one component of vector $g$ is identified. After $N$ iterations equation
$$e(i + N) = g,$$

(8)

is obtained. So independently of the initial set of coefficients $e(i)$ the impulse response of the unknown LTI system is identified after $N$ iterations.

In order to verify above consideration a perfect sequence $p(i)$ and a specially white noise signal were applied as input signal $p(i)$ to the system given in Fig. 1. The results are summarized in Fig. 2 in terms of the logarithmic system distance $\Delta_{ij}$, which is introduced as a measure of quality according to
$$\Delta_{ij} = 10 \log \frac{|g - c(j)|^2}{|g|^2}.$$

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Applying the perfect sequence periodically to the input of the system provides the expected optimal results, i.e., perfect sequences represent the optimal excitation signal of the NLMS algorithm. According to Fig. 2 the initialization phase takes $2N$ iterations to achieve perfect adaptation. This delay is caused by the NLMS error filter of the unknown system and by $N$ iterations, which are required to adapt $N$ filter coefficients. Consequently, within the continuous identification process only $N$ iterations are required.

However, especially the comparison with the system identification achieved with a white noise excitation as e.g. used in [5], [6] visualizes the advantage of the proposed results in a favorable performance for amplitude limited systems.

Ispat sequences [11], [12] are symmetric ternary sequences with a perfect periodic autocorrelation function according to eq. (5) and represent one class of sequences with generally high energy efficiency according to [10] with
$$\eta = \frac{\sigma_w^2 - \sigma_c^2 - 1}{\sigma_c^2 - 1}.$$

(11)

Sequences with even higher energy efficiency are the so called odd-perfect sequences [8], [9], Odd-perfect sequences are symmetric ternary sequences with only one single zero. For this reason the energy efficiency of these near-binary sequences with
$$\eta = \frac{N - 1}{N}$$

is especially high. A synthesis of odd perfect sequences is possible for all lengths $N = q + 1$ with $q = p^k, p > 2$ prime, $k \in N$.

For the system identification approach the odd-perfect sequence is periodically applied as stimulus signal onto the system given in Fig. 1. However, in contrast to perfect sequences the sign has to alternate in each period according to
$$p(i) = (-1)^{i/N} p(i)$$

(13)

in order to obtain an optimal excitation signal for the NLMS algorithm. For this special excitation signal the odd autocorrelation function is given by
$$R_{pp}(i) = \sum_{k = 0}^{N-1} p(k + i) = \begin{cases} |p(i)|^2, & \text{for } i \equiv 0 \mod 2N, \\ -|p(i)|^2, & \text{for } i \equiv 1 \mod 2N, \\ 0, & \text{else}. \end{cases}$$

(14)

Due to Section 2.1 optimal identification of the unknown LTI system is achieved, if $N$ consecutive input vectors $p(i), p(i + 1), \ldots, p(i + N - 1)$ are orthogonal in the $N$-dimensional vector space. A sufficient condition is that the periodic or odd autocorrelation function vanishes for all out-of-phase values for $i = 1, \ldots, N - 1$ condition.

Consequently, odd-perfect sequences fulfill the requirement of an optimal orthogonal excitation and can be applied within the system identification approach.

Fig. 3 shows the comparison of the identification process applying an Ispat sequence of length 121 and an odd-perfect sequence of length 122, each of same maximum amplitude. The corresponding energy efficiencies can be determined to
$$\eta_{Ispat} = \frac{81}{121}, \quad \eta_{OD} = \frac{121}{122}.$$n

(17)

According to Fig. 3 the higher energy efficiency results in a better system identification. For this favorable example the gain amounts to -1.7 dB. For this reason the
following simulations exclusively odd-perfect sequences are applied.

Due to the theoretic analogy of perfect and odd-perfect sequences in this application from now on we will not distinguish between $\hat{p}(i)$, $\hat{R}_{pp}(i)$ and $\hat{p}(i)$, $\hat{R}_{pp}(i)$. All following discussions hold for perfect and odd-perfect sequences.

2.3 Influence of Noise

In this section the disturbance of the adaptation process caused by the noise signal $n(i)$ is discussed. For a spectrally white noise signal $n(i)$ and a perfect (or odd-perfect) sequence excitation $p(i)$ the steady-state level of identification can be determined to

$$D(\infty) = 10 \log \frac{E\{n^2(i)\}}{E\{y^2(i)\}}. \quad (18)$$

The proof of this equation can be performed analogous to [4], where eq. (18) was developed for a stochastic white noise excitation signal.

For an investigation of this relation Fig. 4 depicts the corresponding simulations for various power ratios. One can easily verify the correspondence of the simulation results with eq. (18).

2.4 Influence of Stepsize

Within the last section the influence of the noise signal $n(i)$ was discussed. According to [4] the stepsizes $\alpha$ may also be viewed as the memory of the NLMS algorithm in the sense that it determines the weighting applied to the coefficient update. So the introduction of a smaller stepsizes, $0 < \alpha < 1$, refers to an averaging procedure, which leads to a reduction of the interfering influence.

The theoretic formulation for the steady-state performance in case of a white noise signal $n(i)$ and a stochastic white noise excitation signal is presented in [4] according to:

$$D(\infty) = 10 \log \frac{E\{n^2(i)\}}{E\{y^2(i)\}} + 10 \log \frac{\alpha}{2 - \alpha}. \quad (19)$$

Applying a similar derivation it can be shown that eq. (19) also holds for a perfect sequence excitation of the NLMS algorithm.

The effect of the stepsize may also be viewed in Fig. 5. The comparison of the curves referring to different stepsize factors indicates the improvement with decreasing stepsize value. However, besides the gain of steady-state performance Fig. 5 also illustrates a decreasing convergence speed.

An adaptive stepsize control mechanism, such as used in the application of acoustic echo control [13], represents a concrete approach in selecting an appropriate compromise, see also dashed line in Fig. 5.

2.5 Influence of Sequence Period

So far the two filters $g$, $c(i)$, as well as the period of the perfect sequence were of same length $N$. In this section the influence of the sequence period, especially in case of mismatched lengths, is discussed. For this reason the constants $N_g$, $N_c$, $N_p$ are introduced to distinguish between the length of the unknown impulse response $g$, the filter length $c(i)$ and the period of the perfect sequence, respectively.

As a result of Section 2.1 the period of the perfect sequence length should equal the length of the adaptive filter as well as the length of the unknown impulse response, i.e. $N_p = N_c = N_g$. In this case, assuming $\alpha = 1$ and $n(i) = 0$, optimal identification is achieved, see Fig. 2 and Fig. 6. For an actual realization the first condition $N_p = N_c$ can easily be considered, while the length $N_p$ is normally unknown, so that the second condition $N_p = N_g$ is difficult to meet.
following simulations exclusively odd-perfect sequences are applied.

Due to the theoretic analogies of perfect and odd-perfect sequences this application from now on we will not distinguish between \( \rho(1) \), \( \beta(1) \) and \( \rho(1) \), \( \beta(1) \). All following discussions hold for perfect and odd perfect sequences.

### 2.3 Influence of Noise

In this section the disturbance of the adaptation process caused by the noise signal \( n(t) \) is discussed. For a stationary white noise signal \( n(t) \) and a perfect (or odd-perfect) sequence excitation \( \beta(t) \) the steady-state level of identification can be determined to

\[
D(\infty) = 10 \log \left( \frac{E[N^2(t)]}{E[P^2(t)]} \right)
\]

The proof of this equation can be performed analogously to \([4]\), where \( \alpha \) is introduced for a stochastic white noise excitation signal.

For an investigation of this relation Fig. 4 depicts the corresponding simulations for various power ratios. One can easily verify the correspondence of the simulation results with eq. (18).

### 2.4 Influence of Stepwise

Within the last section the influence of the noise signal \( n(t) \) was discussed. According to \([4]\) the stepwise \( 0 < \alpha < 1 \) refers to an averaging procedure, which leads to a reduction of the interfering influence.

A theoretical formulation for the steady-state performance in case of a white noise signal \( n(t) \) and a stochastic white noise excitation signal is presented in \([4]\) according to

\[
D(\infty) = 10 \log \left( \frac{E[N^2(t)]}{E[P^2(t)]} \right) + 10 \log \frac{\alpha}{\beta - 2 - \alpha}
\]

Applying a similar derivation it can be shown that eq. (19) also holds for a perfect sequence excitation of the NLMS algorithm.

The effect of the stepwise may also be viewed in Fig. 5. The comparison of the curves referring to different stepsize factors indicates the improvement with decreasing stepsize value. However, besides the gain of steady-state performance Fig. 5 also illustrates a decreasing convergence speed.

An adaptive stepsize control mechanism, such as used in the application of acoustic echo control \([13]\), represents one concrete approach in selecting an appropriate compromise, as also dashed line in Fig. 5.

### 2.5 Influence of Sequence Period

So far the two filters \( g(1) \), \( c(t) \), as well as the period of the perfect sequence were of same length \( N \). In this section the influence of the sequence period, especially in case of mismatched lengths, is discussed. For this reason the constants \( N_p = N_p \), \( N_p \) are introduced to distinguish between the length of the unknown impulse response \( g(t) \), the filter length \( c(t) \) and the period of the perfect sequence, respectively.

As a result of Section 2.1 the period of the perfect sequence length should equal the length of the adaptive filter as well as the length of the unknown impulse response, i.e. \( N_p = N_p = N_p \). In this case, assuming \( \alpha = 1 \) and \( n(t) = 0 \), optimal identification is achieved, see Fig. 2 and Fig. 6. For an actual realization the first condition \( N_p = N_p \) can easily be considered, while the length \( N_p \) is normally unknown, so that the second condition \( N_p = N_p \) is difficult to meet.

### 2.6 Tracking Properties

Based on the capability of perfect sequences in combination with an NLMS-driven algorithm to identify the unknown system within \( N \) iterations, the proposed concept is able to efficiently track the fluctuations of a time-variant impulse response.

To visualize the tracking properties of the new identification algorithm a synthetic room model has been developed on the basis of numerous measurements in a strongly time-variant room. Fig. 7 outlines the comparison of the curves referring to the time fluctuation of the synthesically generated coefficient and the tracking behavior of the identification algorithm.

As a result of its tracking properties the identification algorithm provides in every single iteration step a close approximation of the actual unknown system, e.g. a room impulse response. So the proposed identification approach can be used for the simulation of time-variant room impulse responses, see \([15]\) for more details.

### 2.7 Complexity

The identification process bases on the NLMS algorithm. So in each time instant \( M \) multiply and add operations are required for filtering and coefficient update, respectively. In order to store the filter states and coefficients \( 2N \) storage locations are needed. However, due to the use of ternary sequences the multiplicity operations for the coefficient update as well as the storage locations for the filter states, i.e. the perfect sequence, can significantly be reduced.

### 3. Conclusions

In this contribution a new identification approach has been introduced, which is based on an NLMS-driven al-
algorithm excited by perfect sequences. Due to the special orthogonal properties of perfect sequences the NLMS algorithm is capable to identify an unknown impulse response within $N$ iterations. The complexity needed for the on-line identification of the impulse response is significantly lower than for an NLMS-type adaptation algorithm. The properties and performance of the identification approach have been theoretically analyzed and all results were confirmed by simulation.

With the odd-perfect sequences a particularly favorable class of sequences for this application is used. Besides the high energy efficiency the odd-perfect sequences are available for numerous lengths.

Beyond the possibility to apply the proposed concept as measuring system for an unknown transfer function, due to its generality it can be employed in many other applications such as acoustic echo control, acoustic feedback control, active noise reduction or channel estimation.

For instance, exciting the NLMS algorithm in an acoustic echo control application briefly with an audible perfect sequence leads to an excellent initialization of the compensator. Also combined concepts such as proposed in [16], [17] or [18] lead to an improved performance of the considered system. As a result of its convergence speed the identification algorithm is able to track the fluctuations of a time-varying impulse response. Therefore, consecutive sets of coefficients can be used to simulate the time varying characteristic e.g. of a room in an acoustic echo control application, see [15].

Since the NLMS algorithm is already used in many of these applications, the system identification approach can be implemented without the expense of an increased computational complexity or a high programming effort.

References


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